Statistical Signal Processing
Major Test: (7.4.2016)

Time: 2 hours Max. Marks: 40

1. A random process $x(n)$ is known to consist of a single sinusoid in white noise

$$x(n) = A sin(n\omega_0 + \phi) + w(n)$$

where $w(n)$ has a variance of σ_w^2.

(a) Suppose the first three values of the autocorrelation sequence are estimated and found to be

$$r_x(0) = 1; \quad r_x(1) = \beta; \quad r_x(2) = 0$$

Use these values to estimate the variance of the white noise, the frequency of the sinusoid, ω_0, and the sinusoid power, $P = \frac{1}{2}A^2$ via the Pisarenko harmonic decomposition. (6)

(b) How does the estimate of the noise variance depend on β? Does the estimate of the sinusoid power depend on β? Explain the reasons for these behaviours. (2)

2. Consider the AR process

$$y(n) + a_1 y(n-1) + a_2 y(n-2) = w(n)$$

where $w(n)$ is a white noise process of zero mean and variance σ_w^2.

(a) Draw a schematic diagram for a second order adaptive predictor with time-varying coefficients $h_1(n), h_2(n)$ operating on the appropriate samples of $\{y(n)\}$, predicting $y(n)$ from its two past samples, in which these coefficients are adapted via the gradient descent algorithm.(2)

(b) Write an expression for the mean square prediction error σ_y^2 in the steady state. (3)

(c) Write an expression for the 2×2 correlation matrix $\mathbf{R}$ of the tap signal vector. (3)

(d) Write an expression for the eigenvalue spread of $\mathbf{R}$. Why is it important? (2)

3(a) Consider the $p'th$ order autoregressive sequence $y(n)$. Show that for such a process the projection of $y(n)$ onto the space spanned by the entire past $\{y(n-i); 1 \le i \le \infty\}$ is the same as the projection of of $y(n)$ onto the space spanned only by the past p samples $\{y(n-i); 1 \le i \le p\}$
. (4)

(b) Derive an expression for the Burg method for estimating the reflection coefficient for the $i'th$ stage of a lattice filter based on minimisation of the sum of the forward and backward prediction residuals of the $i'th$ stage. Are there any assumptions involved in deriving this expression? If so, clearly state these assumptions. (5)

4. Consider the problem of noise cancellation for the following system:

$$x(n) = d(n) + v_1(n), \quad y(n) = v_2(n), \text{ where}$$
$$d(n) = sin(\omega_0 n + \phi)$$
$$v_1(n) = a_1 v_1(n-1) + v(n)$$
$$v_2(n) = a_2 v_2(n-1) + v(n)$$

where v(n) is zero mean, unit variance white noise, and ϕ a random phase independent of v(n). This ensures that $d(n)$ is uncorrelated with $v_1(n)$ and $v_2(n)$.

(a) Show that

$$R_{xy} = \frac{a_1^k}{1 - a_1 a_2} \quad k \geq 0$$

$$R_{yy}(k) = \frac{a_2^k}{1 - a_2^2} \quad k \geq 0$$

(6)

(b) Draw a schematic diagram for estimating $d(n)$ via cancellation of $v_1(n)$ in $x(n)$, by using y(n) as a reference. (2)

(c) Show that the infinite order Wiener filter for estimating $x(n)$ on the basis of y(n) has a (causal) impulse response

$$h_0 = 1, h_k = (a_1 - a_2) a_1^{k-1} \text{ for } k \geq 1.$$

(5)